The Little Gauss

What most probably happened?

There is this famous story about Carl Friedrich Gauss (1777-1855), the famous mathematician. Already when he was a young kid, he made a wonderful discovery. His teacher, being angry at the class for some petty reason, thought to make the 7-year-old pupils busy for a while by giving them a lengthy and soulless task. They had to add the whole numbers from 1 to 100.

For his disappointment, the teacher got the correct answer from one of the kids very quickly. It was the little Gauss. He enthusiastically blurted out the result, 5050.

Now, this story made me feel uneasy when I heard it for the first time when I was 13, and also later when I was at the grammar school. I couldn't explain, why, but I had a love-hate relationship with this story. I liked the elegance of Gauss' computation, but I couldn't stand the way the wonderful nature of his quick solution was emphasised.

That's why I'd like to reformulate the story in a way that makes sense and can inspire people. So, here's an alternative way to describe what's happened.

When left alone in his room, the little Carl liked playing with numbers. He didn't need anything else, just his imagination. Sometimes he wasn't even aware of these games, but they were so simple and entertaining that he couldn't help playing.

He was just three or four years old, and he recently heard from his father about addition. 1+1 is 2 and 2+1 is 3. After trying adding numbers by 1 up to 20, he saw that they grow evenly. It didn't matter if he added the 1 to 3 or to 13, the result was one greater than the previous number.

During this exploration, he discovered something that he enjoyed very much. First, he added 1+2, then 2+3, then 3+4, and so forth. He saw that the results are 3, 5, 7, ... Although he didn't yet know the concept of odd numbers, he did understand that the growth of these numbers happens evenly. He was thinking about why this might be the case. It didn't take long for him to realise that 2+3 and 1+2 have a strong relation to each other. When he compared the terms in the addition, it was simple to see. When he "transformed" 1+2 into 2+3, he actually made 2 from 1, and 3 from 2. So, he increased each term by 1. No wonder that the sum 2+3 greater by 2 than the sum 1+2. Similarly, the sum 3+4 greater by 2 than the sum 2+3.

A few days later he found something else. It was again very simple, and still it made him smile. Somehow Carl noticed that 17+3 is 20. This is a nice number: 20 seemed to be big, a lot of unexplored territory was beyond it. 20 had also some connection to 10, which he couldn't explain yet. So, 17+3 was 20. But then he noticed that the same is true for 18+2; it was also 20. Though he found first 17 and 3, there was nothing special about them. Carl could construct 20 also from 18 and 2. And then he realised that there are many such pairs. For instance, 15 and 5 or 12 and 8, or even 4 and 16 (even though this last addition took him much longer to compute).

The next day, he figured out that he could organise the different sums in an orderly manner: 19, 1 and 18, 2 and 17, 3 and 16, 4 and so on. No matter how long he did this, he got always 20. It was not hard to see that the reason is simply because from one pair to the other he took one and added one. Overall, he didn't change their sum: 18 was 1 less than 19, and 2 was exactly one more than 1.

What was the last such pair? It was 1, 19. This startled Carl. This was almost the same pair as the first one, but in reverse order. He got excited. There has to be a reason for this! With some investigation, he noticed that all of the pairs occurred twice.

When he was 7, Gauss already went to school. One day, his class was a real buzz. The teacher couldn't get the kids in order. He tried to enquire them about the times table, but it didn't help. For some reason they were busy with their excitement. Perhaps the weather was so nice, that the kids wanted to play outside.

The teacher decided to teach them a lesson. He wanted them to remember not to be disobedient again and he gave a tedious task. They had to add the numbers from 1 to 100. The teacher was certain to finally be able to sit back for at least half an hour. The kids started working with some discontent.

But Carl got really excited. It occurred to him what he had been busy with in his head much earlier. The pairs! He hadn't thought about them for a long time, and certainly not about pairing numbers beyond 20. But why not try it now? "Let's see! Adding 1 and 19 or 2 and 18 resulted in the same number 20. What if I add 1 and 100, then 2 and 99? Yes, they are the same, 101. And similarly, 3+98, 4+97, 5+96 are all 101. OK, now I have to count all numbers between 1 and 100. The only question is how many pairs there are. That doesn't seem easy.

"How was it with 20? I found that 1 and 19 occurred twice: once it was 19+1, once 1+19. That was the same for 18+2 and 2+18. So, each pair occurs twice. Ah, if I look at the second term in 19+1, 18+2, 17+3, ..., 2+18, 1+19, I see 1, 2, 3, ..., 18, 19. Then there are 19 pairs.

"The same idea with 100 would look like 1+100, 2+99, 3+98, ..., 99+2, 100+1. And it is even easier when it is arranged in this order, because the first term is a counter for the pairs: 1, 2, 3, ..., 99, 100. There are exactly 100 pairs. Each of them is 101. So, the sum is 101 times 100, which is 10100.

"Wait! I see that 2 and 99 occur twice. Oh, no! All numbers occur twice. Then the sum 10100 doesn't make sense. I thought I have the result, but not! Or perhaps?! In fact, each number was added twice: 1, 2, 3, ..., 99, 100. That means that the sum is not correct, yet it is *almost* correct. Namely, it is exactly the sum of all numbers from 1 to 100 counted twice. Then the half of 10100 is the sum of all numbers from 1 to 100: 5050!"

Once Carl got this, he was thrilled. Feverishly, he ran through all the ideas in his mind: pairing, the same sum for each pair, the counter in the pairs, and finally, the fact that each number has been computed twice. He was so excited that he couldn't restrain himself and exclaimed 5050 with joy. This 5050 included all the small ideas that led him to this large but innocent-looking number.

The teacher was angry. Not because Carl revealed the correct solution, but because he was not left alone long enough. In fact, he didn't yet know the result as he'd thought that he had plenty of time to compute it for himself while the kids were supposed to be still busy.

In mathematics, speed is really not important. If someone can be quick at something, that just means that it is not new for them. They have already practiced it and they have strong connections in their brain.

What is important is how people learn and practice. If practicing is a senseless chore, it is not enjoyable and, perhaps more importantly, no strong brain pathways and applicable knowledge is created. In contrast, if your practice is playful and exploratory, you can gain deep understanding. Additionally, when you encounter an analogous situation, you will recognise it, and you can apply what you've discovered during your mental experiments.

The little Gauss was quick not because he was a prodigy, but because he was experienced and he saw the teacher's task as a new, yet familiar situation, with which he could challenge his own knowledge.

By leaving out what's happening in one's head, the other seems to be a blackbox. In fact, a distant, untouchable blackbox. If we want to inspire people, we should tell stories that make ingenious discoveries reachable and we should show all human endeavours in ways that students can believe to achieve one day.